INDUSTRIAL DESIGN

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(1) *(Opposite) Ring assemblies representing 4 complete electron shells of an imaginary atom, according to the model presented in this article.*

A DESIGN FOR THE ATOM:

Physics is in need of a three-dimensional model of the atom to complement the mathematical model provided by wave mechanics, Kenneth Snelson considers. Snelson, who is a structural designer, sees certain similarities between structural principles and atomic theory. Starting from this point, he has developed a model of atomic structure which he considers not only is more rational than conventional models, but also suggests explanation for certain previously unexplained aspects of the quantum theory. Below he argues the case for his model.

BY KENNETH SNELSON

These ideas began with an attempt to understand the symmetry laws for circles arranged on the surface of a sphere. I found that there is a close identity between the characteristics of special circle groups and the quantum numbers which describe the disposition of electrons in atoms.

Out of these relationships I have developed the beginnings of a new model which demonstrates structural reasons for the different types of electronic orbitals, and also suggests why only certain numbers of electrons are present at a given energy level. Shown for the first time is a principle of structure involving the electrons' orbital magnetic fields.

Before introducing the model, I will review briefly the history of atomic science during this century, beginning with the origin of our current image of an atom—the planetary model of Lord Rutherford and Niels Bohr.

The planetary model

The Rutherford-Bohr model of the hydrogen atom, which came into being in 1913, is mechanically analogous to a planetary system, much like that of the earth and the sun. An electron of a very small mass is attracted electrically to a proton with a mass some 1,800 times greater, each one rotating about the other. Since the electron is much lighter it does the greater amount of circulating, moving about this tiny space at nearly 1,800 miles per second—or one per cent of the speed of light.

Although this system was reasonable mechanically, it presented other difficulties. According to classical electromagnetic theory, charged particles when accelerated by electrical fields give off radiant energy in the form of photons. Because of this, a planetary atom should collapse in a fraction of a second by loss of energy.

Niels Bohr therefore hypothesized that there must be stabilized states at which the electron can rotate about the larger particle and not lose energy. His model of the hydrogen atom represents these "stationary states" or energy levels as concentric circular orbits, 1, 2, 3, 4, and so on, at which the system is stable. Energy is given off or absorbed only when the electron jumps from one energy level to another.

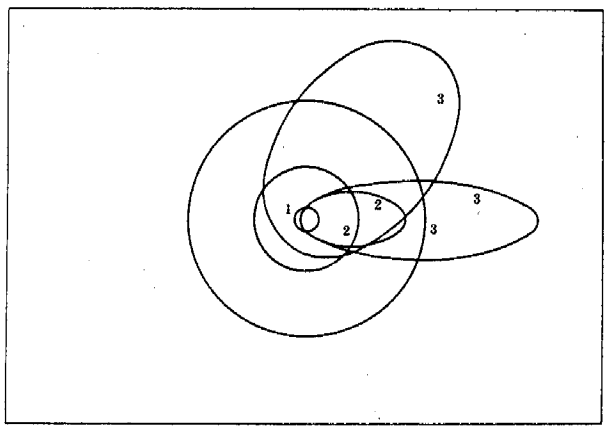
Elliptical orbits

Most information about individual atoms has come from spectra —the light which is emitted by atoms and analyzed by spectroscopists. The first modification of Bohr's atom occurred because of subtle discrepancies seen in spectral lines. Arnold Sommerfeld, a German physicist, determined that Bohr's circular orbits were insufficient to account for particular spectral data. He found that additional types of orbits were required to account for these peculiarities, and therefore prescribed elliptical orbits to supplement the circular ones—ellipses with the same major axis as the Bohr form, but with narrower minor axes (2).

Since the data which Sommerfeld applied to this problem was known in spectroscopy before the atomic model had been devised, the spectral names were attached to the various orbital types: *s* for sharp (lines); *p* for principal; *d* for diffused; and *f* for fundamental— hence, *s*, *p*, *d* and *f* electrons. We still use these letters to identify different orbits.

According to Sommerfeld's modification, the first energy level, or shell, required only the circular orbit or *s* electrons. The second shell had two possible types—a Bohr orbit plus a Sommerfeld ellipse, or *s* and *p* electrons. The third shell required three types, *s*, *p* and *d;* and the forth shell, *s*, *p*, *d* and *f*. This was all remarkably peculiar since each additional kind of orbit carried with it an additional unit of (gyroscopic) angular momentum. But this seemed no stranger than other quantized aspects of the new world of the atom.

(2) *The Bohr-Sommerfeld orbits for shells* 1, 2, *and* 3.



Electron waves

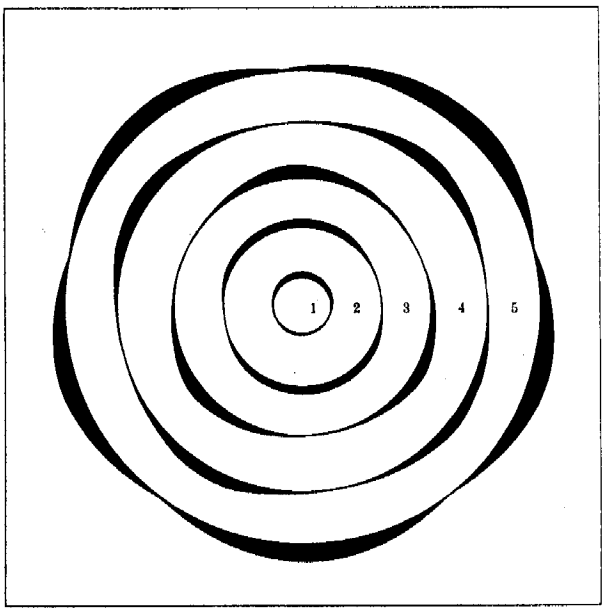
In 1924, a French physicist, Louis de Broglie, postulated that matter, like electromagnetic energy, might also have an associated wave aspect. This was based only on conjecture. Einstein had shown that light waves behave in some ways like particles, so might it not also be true that particles behave like waves? The existence of matter-waves was demonstrated in 1927 by the famous Davisson-Germer experiment, which showed that electrons can be diffracted by crystals in the manner of waves.

To Bohr's atom was joined the de Broglie wave hypothesis, which explained the stationary states as standing wave periodicities. The resulting theory postulates that the electron can settle at only those distances from the nuclear charge where it can fit whole numbers of complete wavelengths. The circumference of the electron's smallest orbit is equal to exactly one complete wavelength. The second orbit has that circumference which can accommodate two complete wavelengths, the third three, and so on (3). The reason for this is that the matter-wave of the electron is a standing wave--it is continuous throughout the cycle of orbital motion — and must therefore be in phase with itself, restating its own pattern at each cycle as if following its own trail. Without the proper circumference to permit this, it would interfere destructively with itself.

Beginning of wave mechanics

Early in 1926, Erwin Schrodinger developed his wave equation which was based on the earlier work of de Broglie and Bohr. Wave mechanics, which resulted from Schrodinger's work, approaches electron interactions in a statistical way. Its difficulties lie in the many numbers of possible interactions in any atomic system involving several electrons. For this reason the hydrogen atom still serves as the prototype for all atoms, and wave mechanics is founded on the assumption that electron waves surround the nucleus much in the manner of a single

(3) *Standing wave patterns for Bohr-de Broglie stationary states.*



electron of hydrogen. When an atom has large numbers of electrons, their interactions can only be dealt with as probabilities. Modifying this picture is Werner Heisenberg's *uncertainty principle*, which describes the limit to which we can be certain about where a moving electron can be located. We can speak only of "charge clouds" which describe the probability of finding an electron in a given volume of atomic space.

The exclusion principle

Since the single electron of hydrogen tends to stay in the first energy level, we might expect the many electrons in heavier atoms to find places there as well. Instead, Wolfgang Pauli's famous *exclusion principle* states that only certain numbers can find residence in each shell.

The exclusion principle, which was defined in 1925, applies to all atoms and describes the order by which the successive energy levels are filled. Basically this principle shows us that electron waves exclude one another from the energy position which each occupies. They are, in this way, similar to larger aggregates of matter which cannot occupy the same space at the same time.

The electrons are given sets of "quantum numbers" according to Pauli's exclusion principle. These provide us with information about electrons somewhat analagous to the information we receive when we purchase theater tickets: mezzanine, third aisle, fourth row, second seat. Similarly, four quantum numbers are required in order to describe the "placement" of any electron in an atom. These four factors determine: (1) the number of the shell; (2) the quantity of angular momentum or orbital eccentricity (like Sommerfeld's ellipses); (3) the direction in which the plane of the orbit faces; (4) the direction in which the electron particle spins relative to its motion in orbit.

Electron spin

Beginning with hydrogen's single electron, we can review the numbers permitted at consecutive energy levels according to Pauli's exclusion principle. Helium follows hydrogen in the table of elements, with a second nuclear charge and a second electron which also resides in the first level. Helium is an inert gas which indicates that its two electrons complete the first shell. This tightly bound system is due to a phenomenon known as "spin pairing." Electron spin was postulated in 1925 and verified experimentally a few years later. It was found that electrons—like many elementary particles—possess an intrinsic top-like quality whether they are at rest or in motion. Spin provides the particle with a fixed quantity of angular momentum and a tiny magnetic field.

As an electron moves in orbit it can spin either in the same direction as its rotation in orbit or in the reverse direction. Helium electrons—or any such spherical (s) pair—have opposed spins. They are pictured as following one another about at opposite sides of the nucleus because of the repelling force of their like electrical charges. They can be thought of as two points on a wheel, which can precess freely as it rotates, describing a spherical volume of space. Because they are always opposite to one another, as if fixed in space, their magnetic spin fields are fixed in relation to one another. In order to reach their minimum energy state, the two spin fields assume a magnetically attractive antiparallel association which helps overcome their repelling energies (4).

When spin was discovered, it was incorporated into the exclusion principle. Until that time it was thought that s electrons were simply two point-charges which moved along the same orbit. But now, s electrons, because their antiparallel spin fields alter each other's courses slightly, are regarded as occupying separate orbits.

Shell number 2 can have a total of eight electrons which are divided into two subshells. Two s electrons form a spherical configuration again; but, unlike the first shell pair, these $2s$ electrons combine chemically with other atoms. This indicates that their paired spins form a looser structure than those of the inner shell. The remaining six electrons of the second shell occupy p orbitals and form the $2p$ subshell. They are not spherically symmetrical charge-clouds, but reflect the elliptical p orbital form described by Sommerfeld (5). As a configuration they are shown to penetrate the inner shells and extend outward from the nucleus along x, y, z axes.

A question frequently raised regarding these orbitals is: how can the p electrons move in and out through the first and second shell s electron spheres without interfering with the stability of the system?

The problem becomes more complex in the third shell, for again a spherical s subshell is formed plus another subshell of six p electrons with their penetrating charge-clouds. Added to this is a third subshell of ten electrons which also penetrate the inner shells. This is the d subshell, or d orbitals which supplant Sommerfeld's second eccentric form of elliptical orbit.

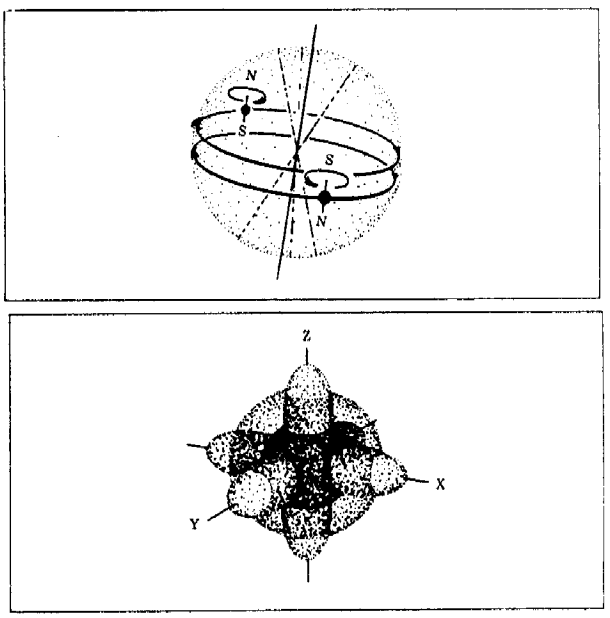
The fourth shell repeats all of the configurations of the third shell but provides for another subshell with fourteen penetrating electrons occupying f orbitals.

An electronic dilemma

In any effort to define a workable system of electron trajectories or wave-paths for a many-electron atom, we are confronted with a dilemma in topology: how can a plurality of matter-waves occupy stationary states and avoid destructive interference? Or, in terms of particles, how can large numbers of electrons moving about the nucleus avoid random perturbations?

(4) Pair of s electrons with antiparallel spins. There is no preferred direction for the axis of rotation, which precesses, making this system spherically symmetrical.

(5) Penetrating charge clouds of a p orbital configuration according to the conventional wave mechanical model. The two spherical forms represent the 1s *and the* 2s *charge clouds.*



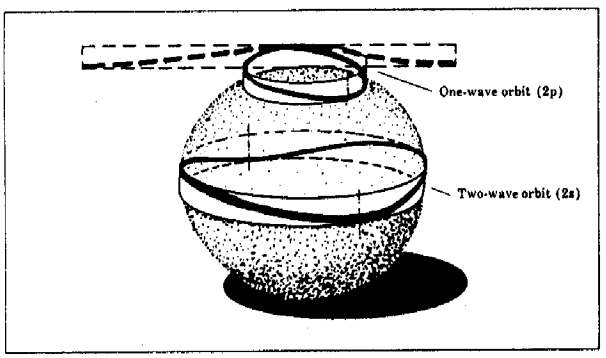
The problem is a mechanical one, for electrons or any charged particles attract or repel one another by producing and exchanging packets of energy, one to the other. If this were to happen at random between electrons of different energies, the entire conception of stable energy levels would be inexplicable and we would not find in spectroscopy the sharp and limited lines which are the fingerprints of the different kinds of atoms.

The difficulty is further intensified in the heavier atoms because, with the increase of nuclear charge, the kinetic energies of the inner electrons become very great. There is no way to explain how the outer electrons, with comparatively small kinetic energies, can repeatedly penetrate the inner shells.

When we look again at the charge clouds of the *p* orbitals (5), we see an image left over from the beginnings of the quantum theory. The underlying assumption is still retained, despite the many changes in the superstructure of atomic theory, that *all* electrons must circumscribe the nucleus in a planetary elliptical manner in order to prevent their falling into it. This would be like an object falling to earth—even though, as we have seen, electrons can "fall" no closer to the nucleus than the first energy level and then only if an electron in the first level has been suddenly removed by collision.

If we are able to take with any seriousness the meaning of the exclusion principle, which indicates that electron waves displace one another as if they were solid objects, then the interpenetrating charge clouds of the *p*, *d* and *f* electrons are completely untenable as structural conceptions.

I will offer an alternative system on which a new model of the atom can be based. It fills the requirements of the quantum theory and presents a logical solution to the collision problem of the interpenetrating charge clouds.



Structural hypothesis for a new model

The electron of the first energy level, according to the Bohr-de Broglie theory, surrounds the nucleus in one standing wave. In this shell there is only the *s* variety of orbit. In the second shell, the electron surrounds the nucleus in two standing waves—and there are two varieties of orbit, *s* and *p*. The third shell electron fits three standing waves into its orbit and, appropriately, there are three types possible. And so on.

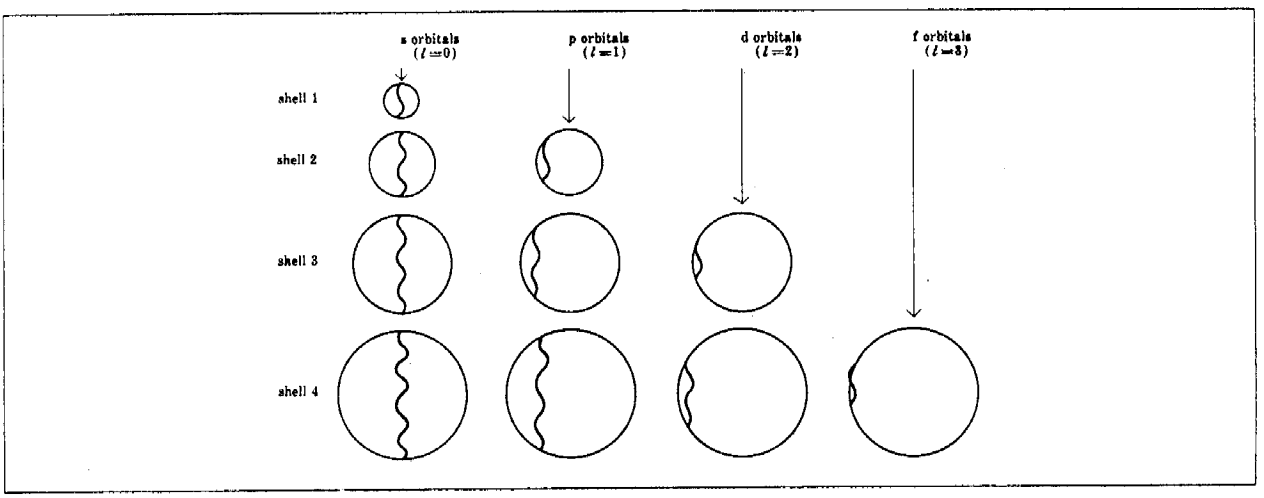
This pattern suggests a connection between the variety of orbital types and the number of standing waves which an orbit contains. Though this may be so, there is no reason to suppose that the different types of orbit in a shell *must* all contain the maximum number of waves for that particular shell.

Suppose, for example, that the second shell electron's two-wave cycle had an alternate orbital form which contained just one wave. The circumference of this orbit would have to be just half the circumference of the standard two-wave orbit. Thus fitting itself into a small circle, the electron could not surround the shell in the same equatorial way—it would be able to follow only a small-circle path on the second shell charge sphere.

We can examine this by means of a device—a billiard ball—to represent the spherical energy surface (i.e. the distance from the nucleus to the second shell.) A strip of paper which just encircles the ball can represent the two wavelengths of the 2*s* electron (6). If we cut the strip in half, each half represents just one of the waves. If we make this single wave into a loop by connecting its ends, it can represent a single stand-

(6) Billiard ball model with paper strips representing a two-wave 2s great circle orbit, and a one-wave 2p orbit.

(7) Schematic chart showing electron energy levels for shells 1, 2, 3 and 4. Reading left to right indicates the number of whole waves, according to this model, contained in the subshells' orbits.



ing wave of the second shell. The loop will not surround the sphere like an equator but will rest on its surface like a small-circle cap. This off-center orbit lies on the same energy surface as the two-wave s orbital but it traverses only a portion of it. The circumference of this spherical segment is equal to one wavelength appropriate to that energy level. Additional energy would be required in order to confine the electron in this way, but an increase of this kind is seen in the fine-structure of spectral lines. The p orbital, which this condition will represent in this model, lies at a slightly higher energy level than the s orbital of the same shell.

A billiard ball model for the third shell, whose s orbitals contain three standing waves, can be represented in the same way. The piece of paper can be cut into a two-wave strip and also into a one-wave strip and made into contracted orbits. Because the two-wave strip is the first squeezing-in of the standard size orbit it must represent the third shell's p orbital type. The one-wave orbit is the final off-center contraction and must become the d orbital type for the third shell.

The fourth shell, which has four-wave s electrons, will have p orbitals with three waves, d orbitals with two waves, and f orbitals with one wave (7).

Structure of an off-center orbit

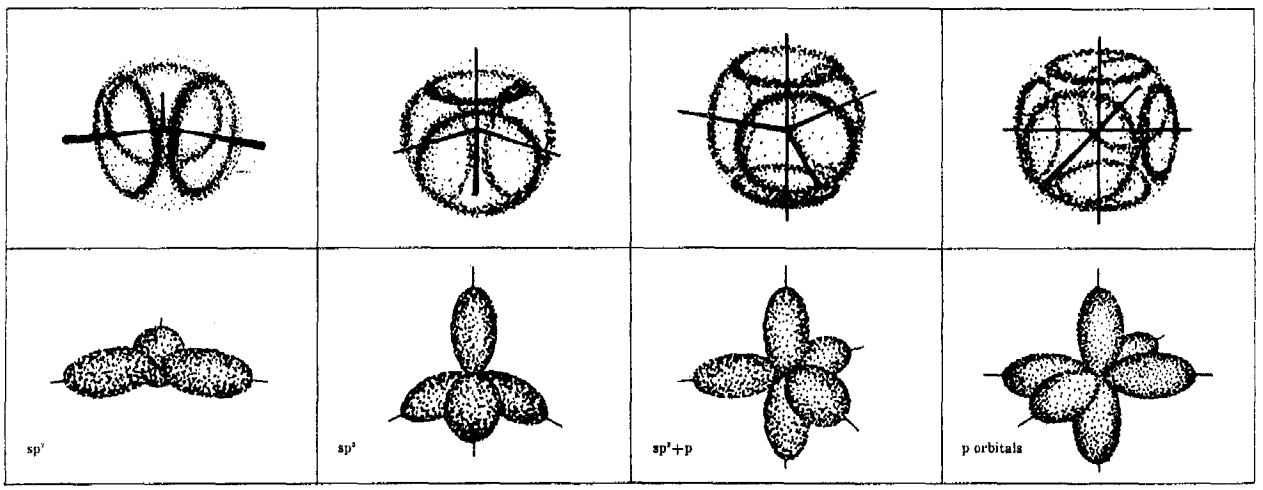
An orbit which does not surround the nucleus centrally, like that of a planet is not easily imagined, at first. It is necessary, however, to keep in mind the restriction of the electron—that it is committed to the formation of a continuous standing wave or else an orbit cannot exist. A wave of this kind could be compared to a cowboy's twirling rope: a loop twirling parallel with the ground will quite normally rotate off center from the hand, like an off-center orbit, due to the pull of gravity on the loop. In the same way, an electrical or magnetic force would be required to deflect an electron's standing wave into an off-center state. In a complex atom such electrical energy is present in the electrons themselves whose repelling charges "lift" one another into these higher orbital states like objects packed in a box.

Several examples of orbital groupings are shown (8, top row)—compared to their counterpart in the conventional model (bottom row). The numbers of electrons and their axial arrangement in the new and standard forms are identical. The difference is the way in which the electrons in the two kinds of diagram occupy space.

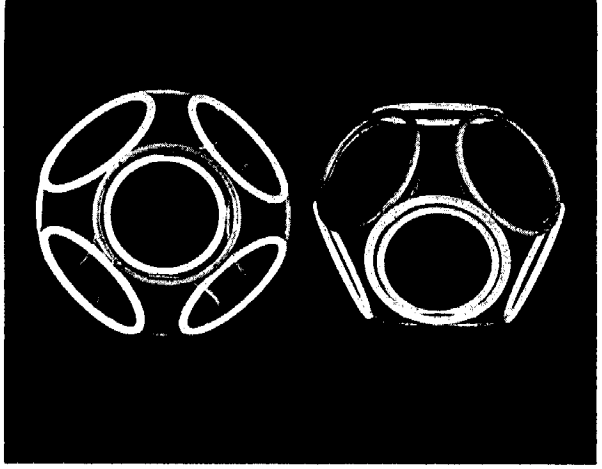
The penetration of the inner shells in the charge cloud model, and the perturbations that might be expected to result, have been discussed. In all of the new forms, however, the 1s shell fully occupies the sphere immediately around the nucleus. The off-center orbits which gather around the nucleus are drawn toward it but prevented from approaching by the electrical shielding of the inner electrons, and also by the repelling force between the off-center charge clouds themselves. Shell upon shell can be added in this way without the electrons' interfering with one another's path of motion. Here then is a structural interpretation of the exclusion principle.

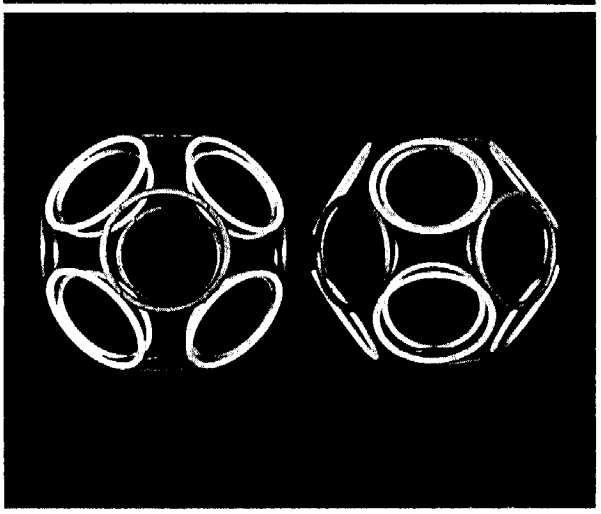
It should be noted that the billiard model we have used is a crude device. In a complex atom, which has shells one outside of another (1), each shell will act as a shield, reducing the effective nuclear charge for the outer electrons. In sequence this will give the outer electrons smaller kinetic energies and longer de Broglie wavelengths, so that the paper strips are not an accurate measure of the relative wavelengths.

The billiard ball model is valuable because it demonstrates a logical alternative to the elliptical charge cloud. It shows that an electron could remain at the same distance from the nucleus—in the same shell—and attain these reduced orbital states. The smaller the orbit becomes, by the removal of whole waves, the more electrons of these types will be permitted around



(8) *Four types of bonding orbitals compared. Top row—new form. Bottom row—conventional form.*





(9) *Model form for the 10-electron d orbitals.*
(10) *Model form for the 14-electron f orbitals.*

an energy surface before they interfere with one another. Thus we find places for 10 *d* electrons and for 14 *f* electrons in these subshells (9, 10).

Angular momentum

Another aspect of the off-center hypothesis concerns the angular momentum of the electron in orbit, which is defined by the *l* quantum number. Spectral analysis shows that angular momentum increases for *p*, *d*, and *f* electrons, in that order. The change in the eccentricity of Sommerfeld's ellipses was meant to account for this observed phenomenon.

This new model accounts for the increase in angular momentum by means of the number of whole waves which the orbit contains. The removal of a wave from a two, three, or four-wave orbit will cause the electron to complete its orbit "one wave" faster, even though its kinetic energy and its actual velocity remain the same. The elimination of a whole wave increases the number of cycles per given unit of time. In terms of the orbit's angular momentum, this has an effect similar to increasing the rate of rotation of a gyroscope. In terms of the orbit's magnetic field, it is the same as increasing the number of coils of a current loop. The quantum numbers show that these forces do increase in this quantized way (by stages) for *p*, *d* and *f* orbitals.

These small-circle systems are more intricately structural than we perceive at first. Orbital angular momentum is a force which stabilizes the orbits like tiny gyroscopes in opposition to one another. Magnetic fields have an equally important meaning in the formation of electron structures.

Magnetic structure

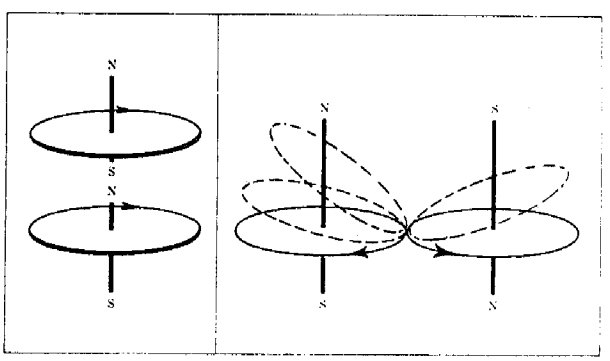
In the earlier description of spin pairing it was shown how pairs of *s* electrons chase one another around the nucleus so that their spin magnetic fields are related one to the other like two permanent magnets fixed in space. If the electrons are not following the same motion course, as they cannot do in the off-center orbits, their spin is smeared and becomes diffused over the entire path of the orbit. It is a thin field, to be sure, but spread out evenly like a blurred image of a time exposure.

Each electron's orbit has a second and equally important magnetic system. It is a separate, orbital magnetic moment, and as its name indicates, it arises out of the orbital motion of the electrical charge. This is identical to the magnetic field of an electric current loop and lies perpendicular to the plane of the orbit, the north pole on one face and the south pole on the other.

There is an important relationship between the two types of magnetic field. As we have seen, the electron can arrange its spin in either the same direction or just the reverse of its rotation in orbit (4). This means that spin can either add to, or counter the orbital field. We will see that this mechanism can work as a cut-off system for magnetism under certain conditions.

If the kinetic energy of the electron is great, as it is for the inner shells of the heavier atoms, the orbital magnetic fields are also very great. In the first shell of

(11) *Current loop magnetic fields attract face to face when their polar directions are identically oriented (parallel); or edge to edge when their poles are reversed to one another (antiparallel)—here they have freedom of angular association, shown by the dotted lines.*



calcium the magnetic moment is about 10^{10} gauss per electron, and in the first shell of uranium about 10^{13} gauss. Forces of this magnitude inside the atomic shells must be dispersed in some way. It would be surprising to find that they have no purpose but to cause perplexing shifts in spectral lines. Yet in all of the considerations given to atomic structure, it is nowhere evident that a useful purpose is expected of these forces. There is much discussion concerning the coupling of the magnetic currents within the atom but no indication of the internal form which this coupling might assume.

I would like to show some experiments, using permanent magnets which have the same shape and polar arrangement as the field of a current loop, to indicate how magnetism may play a major part in the structure of electron configurations. These magnets are made of ceramic material and are shaped like flat washers—polarized on opposite faces like heads and tails of a coin. Flat magnetic fields attract one another in two different attitudes: face to face in parallel, or edge to edge when they are antiparallel (11).

The edge to edge association is of special importance for it represents a relationship like that of the off-center circular orbits of this model, whose edges or wave fringes touch one another. It is possible to assemble groups of these magnets around a sphere in a mosaic of antiparallel fields. But only certain numbers of identical circles can form such closed spherical groups. The only possible sets contain 5, 8, 10, 14, 18 and 32 magnets (12).

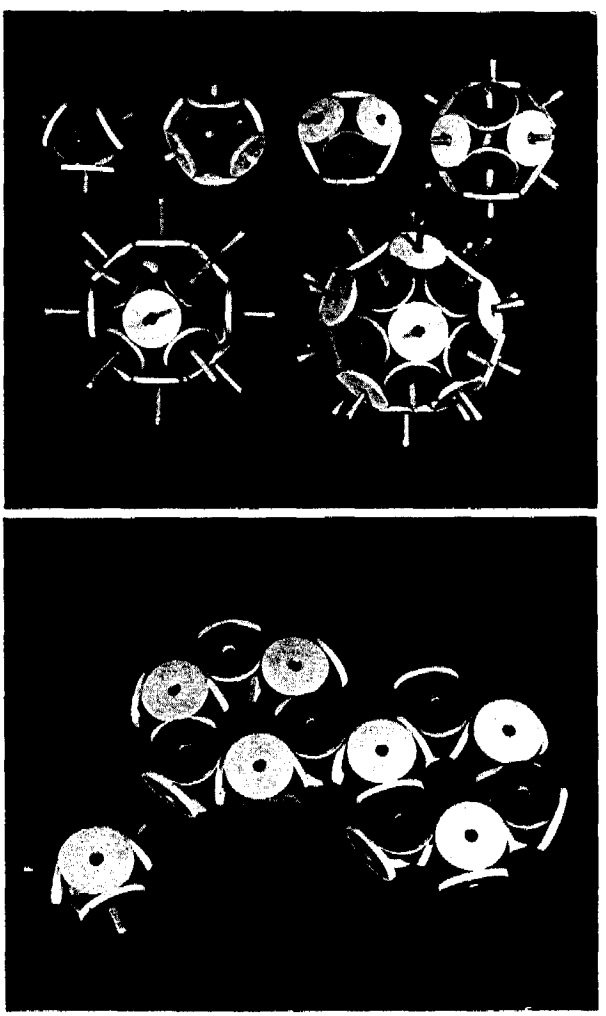
The magnets in these models are placed on non-magnetic armatures in order to support their weight. When they are in the proper antiparallel position they cling together. They can be withdrawn individually and when returned, they are seized by the surrounding neighbors. They are linked together in closed spherical circuits. Because they are patterns of alternating polarity, there is no preferred magnetic direction for the completed spherical forms. This is in agreement with experiments which show that angular momentum and magnetism add to zero for completed shells of electrons. (The usual explanation is that magnetic fields are located randomly in space and therefore cancel one another's effect.)

There is a remarkable correspondence between the numbers of spherical circle groups which can be assembled from magnets and the numbers of electrons in energy levels. The notable exception is the group of five magnets which is the smallest assembly; but this is not to be rejected merely because it does not represent a shell number. Besides forming the sp^2+p hybrid orbitals shown in (8), the group possesses the interesting quality of linking with other groups of five to form hexagon rings with perfect magnetic continuity (13). A group which contains six of the five-magnet forms would seem to require thirty magnets. It is found however that each pair of cells, when joined, can share a magnet which is their common face. This cancels six magnets so that a hexagonal group contains a total of twenty-four. This averages four, rather than five to a cell.

If each magnet is thought of as an electron in orbit the hexagonal assembly becomes an excellent model of the benzine ring of organic chemistry. For here is a flat hexagonal group of atoms which can be repeated in an extended pattern like collections of aromatic ring molecules. As a model, each atom has the proper number of outer electrons—four—to represent an electrically neutral system.

This configuration offers a reasonable explanation for a long debated question of organic chemistry. The

(12) Magnet models containing 5, 8, 10, 14, 18 and 32 magnets. Light and dark faces represent opposite polarities exposed.
(13) Five-magnet model (lower left), and hexagon rings formed from six five-magnet units linked together.



structure of benzine must usually be justified in terms of alternating or *delocalized* bonds in order to account for the proper number of electrons in each carbon atom.

Shells and subshells

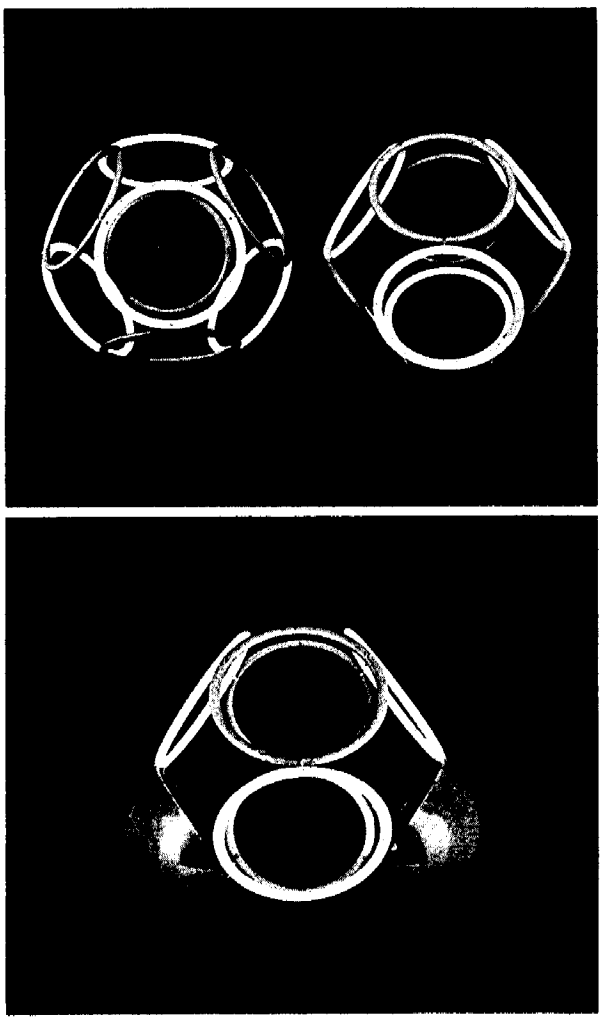
The magnet models do contain the sets of numbers for the completed shells, 8, 18 and 32, as well as the subshell sets of 10 and 14. Absent from the group is the six-circle cubical set which would be expected in order consistently to represent the *p* orbitals. Magnetic coupling is prevented in this arrangement because there is no way in which the six faces of a cube can be checkerboarded to permit the association of opposite magnets. The only completely symmetrical organization for this form is an arrangement like six clocks on the faces of a cube. In terms of magnets, this is a parallel or repelling association. This would add to the electrical repulsion of the electrons and force them into a higher energy state.

If this happens to take place in an outer shell where the kinetic energies of the electrons are small, it is possible for spin to neutralize the orbital magnetic field by reversing its orientation. This would overcome the obstructive magnetic fields, permit the electrons to approach one another more closely and thus reach a somewhat lower energy state.

The electrons in the inner shells have higher kinetic energies and greater orbital magnetic moments. Because spin is a constant it could not serve to neutralize these larger orbital fields. According to this model it is necessary for the *p* electrons in any inner shell to combine with the *s* electrons of that shell forming the single magnetic configuration of eight identical orbits (14). This seems reasonable, since the *s* orbitals beyond the first shell have only a provisional chemical stability and do not represent a tightly closed system.

This eight-circle group is arranged on a spherical surface along the corner axes of a cube. The four circles whose north poles face outwardly alternate with the four whose south poles are exposed. The two sets describe interlocking regular tetrahedra. Although the octet of the inert gases is generally believed to comprise an *s* subshell and a *p* subshell, there is some controversy regarding this. Linus Pauling and others have discussed the possibility that *s* and *p* shells are a single entity like two interlocking tetrahedra. If this question is resolved one day in favor of the single octet shell, then it would support the principle of magnetic linkage presented in this model—see H_2O molecule (15).

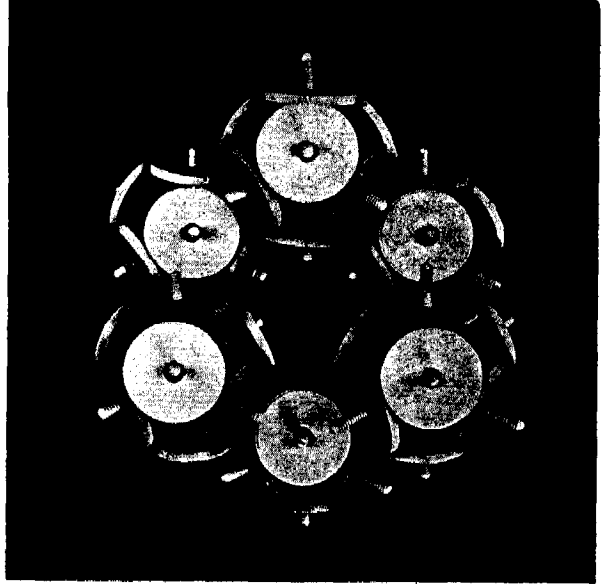
(14) *Eight-circle group which provides for magnetic linkage.*
(15) *H_2O molecule according to this new model. Each half-sphere represents a hydrogen atom bonded to the oxygen atom at 109° 28'.*

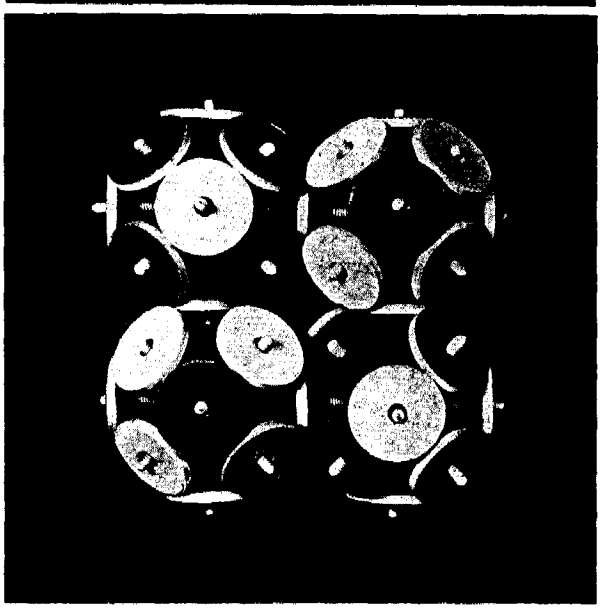


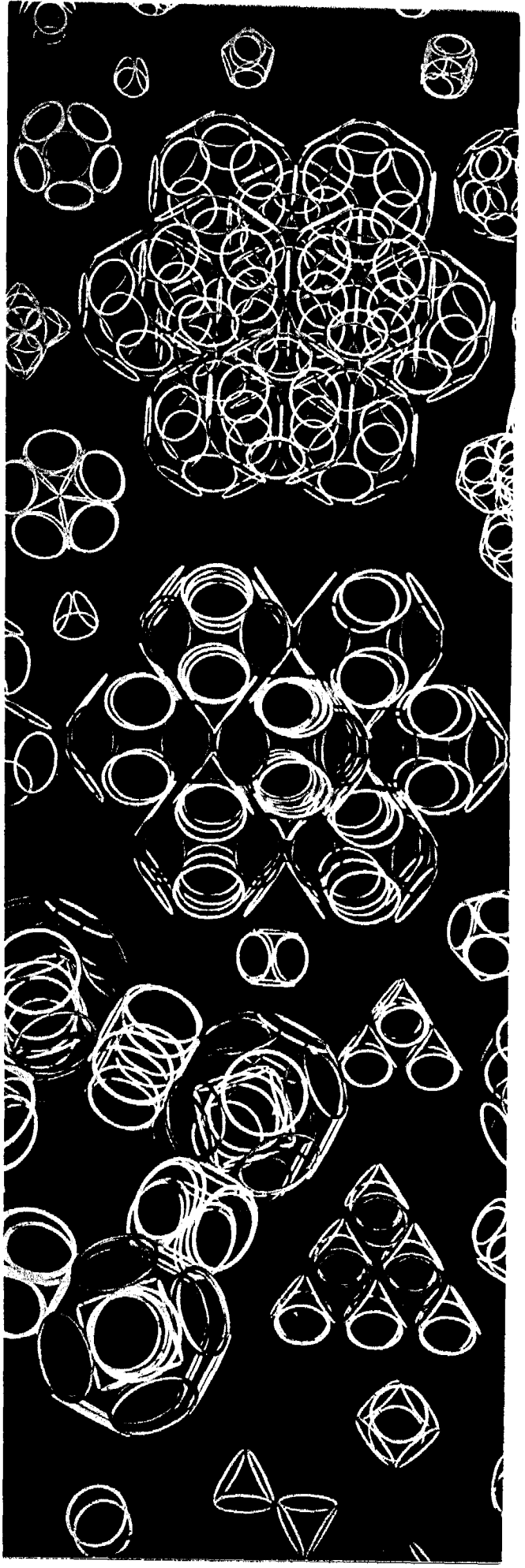
Electronic gearing

A curious quality is found in each of the magnet models: if one of the ceramic wheels is turned by hand, the entire set will rotate, all north poles turning in one rotational sense and all south poles in reverse. For example, in the octahedral set, the two interlocking tetrahedra turn in separate directions. This causes no change in their relationship, since the north and south poles remain constant to one another. The magnets simply act like a set of gears, spherically arranged and in friction contact. This phenomenon, which can be attributed to binary systems (i.e. pairing of the magnets) suggests an extremely important analogy. For what we observe here, in mechanical form, has its exact counterpart in electricity: two orbits or electrical loops must have counter-rotating charges in order to give rise to antiparallel fields.

Perhaps more remarkable is the discovery that each of these magnetic groups can translate in space, cell upon cell like polyhedra with no loss of magnetic or gearing continuity. By turning one magnet an endless







(16) *Eight-circle magnet groups arranged in body-centered cubic form with perfect magnetic continuity throughout.*
(17) *Four groups of the fourteen-circle f orbital forms. The polarities of adjacent groups alternate for magnetic linkage.*
(18) *Models of ring assemblies in various types of order. Numerous combinations can be made by associating different groups.*

assembly could be made to turn like an infinite system of gears. We can see in these forms a structural order of great intricacy—crystalline lattices composed of electron networks magnetically arranged and bound by nuclear charge centers (16 and 17).

Conclusion

These are some beginning notes of a study which is still going forward. I have described the systems in only a general way. There is much to be explored, such as the magnetic interactions between adjacent shells, the relationship of these structures to optical and X-ray spectra, and the variety of ways in which the many figures can describe molecules and crystalline order. Some of the many varieties of circle groups are shown in (18).