TENSEGRITY, WEAVING AND THE BINARY WORLD

NEWTON'S THIRD LAW AND THE DUALITY OF FORCES

by
Kenneth Snelson
Among the terms for “twoness” are duality, binary, twin, pair, couple, double, yin-yang... These words refer to ideas and beliefs in art, literature, religion, science and philosophy in cultures everywhere. Twoness words are associated with good-evil, male-female, north-south, past-future, day-night, up-down, push and pull, life and death...

Structures of many kinds provide evidence of a binary phenomenon rooted in the nature of things, with examples in the clockwise-counterclockwise rotation of cog-gears, of compression versus tension in tensegrity structures, in the right and left hand helices of fabric weaving or simply in the reversing of colored squares of a chess board.

Isaac Newton’s third law of motion states clearly and simply the binariness in physical forces: for every action there is an equal and opposite action. In this picture essay I describe how Newton’s third law and duality are reflected in many kinds of structures.
THE CHECKERBOARD

In checkerboard patterns two colors alternate cell-to-cell. Checkering is a figure-ground design with an aesthetic all its own, a visual system found in the art and architecture in cultures all over the world.

Whether composed of polygons (Fig. 1) or random shapes (Fig. 2) the checkerboard grid displays the primary beauty of binariness where neighbors are of opposite color.

The crossing of two lines, one line passing through another, (Fig. 3) is a phenomenon, a first-principles event that initiates a checker pattern. The intersection where the lines cross divides the plane into quadrants which allow the binary coloring of alternate cells.
CHECKERED SCRIBBLES

A scribble of closed loops, Fig. 4, allows for two-color checkerboarding: Each locus where lines intersect establishes four distinct areas of alternating colors.

In Fig. 5 small red “bridges” are superimposed at each line-crossings (magnified in Fig. 5a) As in woven fabric the crossings alternate over-and-under throughout the squiggle figure. The looping line with its many crossings can be viewed as a woven knot. This beautiful phenomenon exemplifies the essential binariness of nature.
Because the checkerboard principle concerns alternating of neighbors it applies also to the binary clockwise / counterclockwise rotation of gear trains (Fig. 7) and the north pole/south pole attraction of magnets with polarity on opposite faces: north on one face, south on the other. (Fig. 8 and 8A)

**Fig. 7**

**Fig. 8**

**Fig. 8A**

Disk shape magnets and current loop magnets attract edge-to-edge when poles are opposite. Or face-to-face when north and south poles are in alignment.
ABOUT WEAVING

The ancient invention of weaving displays the basic properties of natural structure: modular-repetition, left and right helical symmetry and the close association between geometry and physical structure.

Two and only two fundamental fabric weave structures exist: the standard two-way plain weave made up of squares, Fig. 9, and the three-way triangle/hexagon weave, Fig. 10, used most often in basketry. Though there are many variations such as criss-crossing, doubling, etc. these two are the only primary forms.

A single weaving event, two filaments crossing and in contact with one another, Fig. 11, each warping the other where they press in contact is, in itself, an elementary structure. At the point of crossing the two threads create dual helical axes one clockwise, right-rotating and the other counterclockwise, left-rotating.

Bypasses, crosses and Xs have become powerful symbols and signs: a christian cross, skull and crossbones, crossed fingers, Xd out, sign here, keep out.

This elementary crossing principle reflects the checkerboarding scribble’s intersections of Fig. 5. This binary property along with the right and left hand rotation of gears Fig. 7, the north and south polarity of magnets Fig. 8, are the foundation of binariness. These dualites, these reversals of right and left rotation at every crossing, provide nature’s first lesson in fundamental structure. The helical phenomenon plays a vital role in determining whether two separate parts will or will not link together.

---

Cross two pencils. Place a thumb and index finger on the pencils and slide toward the center. Your hand will tend to rotate either clockwise or counterclockwise.
RIGHT HELIX, LEFT HELIX

FIG. 10. SQUARE WEAVE
Just as the individual crossings of filaments have alternating helical axes so each square in a plain weave alternates with its neighbors like chess board squares. In order to prove whether a weave cell is right or left handed, imagine your fingers sliding in contact with the frame of a cell. Your hand will move “down-hill” in a clockwise/counterclockwise sense according to the cells “rotation”.

FIG. 11. KAGOME THREE-WAY BASKET WEAVE
In three-way, or Kagome weaving, hexagons alternate with triangles. If the hexagons have a clockwise helix the triangles are counterclockwise. If the hexagons are counterclockwise the triangles are clockwise.
THE FIVE BASIC WEAVE CELLS

Below are the five basic weave cells. The five-way pentagon is used only in basket-weave spheres or compound-curvature baskets.

- Two-way cross unit
- Three-way triangle unit
- Two-way plane weave unit
- Five-way pentagon weave unit required in basket-weave spheres
- Three-way hexagon weave unit

Rattan basket-weave ball with outlined triangle, pentagon and hexagon cells. Thailand

Carved ivory ball with basket-weave pattern. China, 19th century
WOVEN COLUMNS

The three columns shown here share an identity with braiding or plaiting. The struts of the tensegrity column (E. Q. Tower) have a weave pattern although they are not directly connected to one another.
The x-module column also is defined by weaving. On the left is a floating compression two-way or "x-module" column shown with blue connector paths superimposed to identify the compression members’ weave paths. The figure on the right is a vinyl tubing woven column section that requires four interwoven tubes to emulate the tensegrity x-module pattern. The expression “two-way” refers to each x-module’s set of two struts.
The weave cells shown so far relate to polygons; to triangles, squares, etc. with edges that bypass one another. It is possible also to translate three-dimensional solids, tetrahedra, octahedra, etc., into weave-like cells by using sticks as the polyhedra’s edges. I call these hybrid configurations “weave-polyhedra”. Shown below: a weave-tetrahedron, a weave-truncated-tetrahedron, a weave-octahedron and a weave cuboctahedron. Because of the helical bypass at their corners these three-dimensional structures have all the characteristics of the fabric weave cells except that each one is a spatial figure like its parent polyhedron.
The art of weaving has existed since the beginning of civilization. Archeologists have turned up evidence of weaving in Egypt as early as 4,000 BC. To have invented this universal craft must have been indeed astonishing.

In 1965 I was experimenting with modularly repeated tensegrity systems when I began to understand that there is an unmistakable family-connection between tensegrity and weaving.

Above is a photo of me in Sagaponack, New York, with my first dowel stick constructions of tetrahedral (on the left) and octahedral (on the right) space frames. A third form (in the center) composed of cubes lacks triangulation which disqualifies it as a stable space frame. Whether my discovery was truly novel or merely a rediscovery of something known earlier, perhaps in another age in another civilization, is impossible to know. The U.S. Patent office was unable to turn up any earlier example. #6,739,937

Three-dimensional weaving can be seen as an extension of conventional flat weaving. In 3D weaving the two-way and three-way flat planes are made to criss-cross in orderly ways that give rise to “weave-polyhedra” as described on the following pages.

A likely reason spatial weaving hasn’t been discovered or practiced is that, while there are endless uses for fabric and basket weaving, there has been no necessity for space-frame weaving.

A Korean engineer, Ki Ju Kang, is developing an application for 3D space frames. He has been experimenting with three-dimensional weaving he has named, “Wire-woven bulk Kagome” trusses. Ki Ju hopes his planar wire trusses will be adapted in manufacturing high-strength steel sandwich-panels for ship building and aircraft technology.
OCTAHEDRON/CUBOCTAHEDRON WOVEN SPACEFRAME

Four weave-octahedra, illustrating the basic three-dimensional octahedron/cuboctahedron weave pattern. In the center of the group is a square that identifies the cuboctahedron; only half complete in this four-cell example.

Rattan octahedron/cuboctahedron woven spaceframe
TETRAHEDRON
SPACEFRAME WEAVE

This spatial weave pattern has four different directions of triangle/hexagon flat weave planes. These repeated planes align with the face planes of a normal tetrahedron. The alternate, larger forms, the weave-truncated-tetrahedra, occupy the cavities in between the weave-tetrahedra.

Five weave-tetrahedra, showing the basic three-dimensional tetrahedral weave pattern composed of weave-tetrahedra and weave-truncated-tetrahedra alternating in space with one another.

Rattan tetrahedron/truncated-tetrahedron woven spaceframe.
FROM WEAVING TO TENSEGRITY
Weave and tensegrity share the principle of alternating helical directions, of left-to-right, of bypasses clockwise and counterclockwise.

In the top row above are five primary weave figures. Below them are the equivalent tensegrity modules. Individual tension lines -- strings, wires or rope -- are attached to the ends of the struts as shown so that each assembly is a closed system made of tension and compression parts. Each tension line connects individually to the ends of two struts. They do not thread through like a string of beads. The tension lines must be adjusted for tightness as with tuning a stringed instrument or inflating a car tire.

Tightening the tension system stores both tension and compression forces in equal amounts, a state that engineers call “prestressing.” The energy remains stored inside the structure until it is disassembled.

In the figures above, only the 2-strut “x-module” and the 3-strut prism have tension networks with total triangulation. The networks of the square prism, the pentagon prism and the hexagon prism are not composed of triangles. In tensegrity structures triangulation in the tension network is significant because it determines if the structure will be firm or not.

Tensegrity structures are endoskeletal, as are humans and other mammals whose tension “muscles” are external to the compression members’ bones. Unique to tensegrity, the compression struts are separated one from another, non-touching within their tension envelope. The exception is the two-strut x-module, or traditional kite frame. This essentially flat figure lacks a compression force in the “z” direction. In order to separate the crossed struts a third strut or else an additional X-module, must be added to pull the two struts apart.
WEAVE POLYHEDRA AND TENSEGRITY POLYHEDRA

WEAVE TETRAHEDRON; THE VERTICES (RED) HELICES ARE CLOCKWISE. THE FACES (BLUE) ARE COUNTERCLOCKWISE

WEAVE OCTAHEDRON; SAME AS ABOVE: THE VERTICES (RED) HELICES ARE CLOCKWISE. THE FACES (BLUE) ARE COUNTERCLOCKWISE

TENSEGRITY TETRAHEDRON TOTAL TRIANGULATION

TENSEGRITY OCTAHEDRON; TRIANGULATED NETWORK EXCEPT FOR ITS SIX SQUARE FACES
RIGHT HELIX / LEFT HELIX
CONNECTING MODULES TOGETHER

A right-handed tensegrity module can be transformed into a left-handed one but it must be completely reconstructed, exchanging all parts in relation to one another starting with the twist relationship of the compression struts. The right and left configurations are mirror images of one another. What are the consequences of this reversibility? It is that in the mirrored figures, the directional sense of all prestressed pull-and-push forces are also reversed. The tension forces that pull counter-clockwise in a left-handed form pull clockwise in a right-handed form and vice versa.

In column-structures there is an advantage to alternating helical directions module-to-module because the inherent flexibility of a tensegrity structure is in itself helical. When pressed on, a right-handed module rotates slightly to the left and vice versa, so that the entire tower structure flexes when compressed top-to-bottom. By alternating modules right-left-right-left helical rotation in the column comes to zero.
It is possible to construct any number of varied tensegrity configurations, from simple to highly complex. Yet, only those forms whose tension network is composed entirely of triangles are truly stable. If the network has squares, pentagons, etc. the structure will lack firmness. This is especially true of tensegrity spheres, none of which have triangulated tension networks.

The triangles in a tensegrity network are formed in two different ways, designated as type 1 and type 2 triangles.

**Type 1: TENSION/COMPRESSION TRIANGLE.**

Working much like a sling used by riggers for hoisting, triangles of type 1 are formed with two struts and two tendons. The two tension lines run from the end of one strut to the two ends of a second strut.

**Type 2: TENSION-ONLY TRIANGLE.**

A tensegrity triangle can also be formed with three tension lines attached to three different struts.

The two different triangle types identified on a photo of *Needle Tower* at the Hirshhorn Museum and Sculpture Garden in Washington, D.C.
Because all tension lines, strings, wires, cables, have some degree of elastic stretch, tensegrity structures themselves are elastic and springy depending on the tightness of the prestressing and the characteristics of the tension material.

The elastic flexing of a tensegrity structure, a column for example, can be seen in the small rotations of the right or left helixes. A right-handed helix compresses with left rotation and vice versa.

The tower sculpture shown here is as an excellent example. All tension lines -- edges, slings, draws -- are of equal length so that all type 2 triangles, colored green in the picture, are equilateral. When pressed down on, and then released, the column responds like a coiled spring. Its name is *Equilateral Quivering Tower.*
FOLD LINES OF TYPE 1 TRIANGLES.

Type 1 triangles occur always in pairs like butterfly wings. The fold line of each wing-pair of triangles is similar to a crease in the folded-paper column to the right. The springiness of tensegrity structures happens in the hinging along the “fold lines”.

Kite frame structure with its two struts and four tension lines is composed only of type 1 triangles. Type 1 triangles occur here also in pairs making a diamond form. In the kite frame, two face-to-face diamonds share the tension lines on opposite faces of the kite. Note that this most economical of structures is actually a flattened tetrahedron.

This folded paper column simulates the geometry of a three-way tensegrity column. The type 1 triangles are red and the type 2 are in green. Unlike tensegrity, this paper facsimile is not a prestressed structure. Still, it seems likely that the valley and hill folds even of randomly crinkled paper bear a close relationship to tensegrity’s tension and compression patterns of force.
THREE TYPES OF TENSION LINES

A tensegrity structure's whole tension network is external to the compression struts so that it is an endoskeletal structure with compression forces pushing out against the tension skin.

Each separate line connects two points edges, draws and slings and each type plays a specific role in the tension network.

*Edge* tension lines define the edges and the sides of each module. The three-way column has three edge lines for each module. In most cases, edges carry less tension than draw or sling lines.

*Draw* tension lines pull the modules toward one another. In the three-way column, each module is connected to each neighbor by three ascending draws and three descending draw lines.

*Sling* tension lines suspend the modules, performing like the slings used in rigging work. They connect one module to the next and are generally in opposition to the draw lines. In a three-way column six slings are required in order to link two modules.

To the right is a three-way module. In this simple figure all tension lines can be called edges: six end-edges and three side-edges. They define, roughly, a triangular prism. When viewed through the vertical axis the module has a left-rotation helix. The opposite is true when viewed from the side: the struts relate to one another in a clockwise or right-rotation helix.
The simple kite frame, two crossed struts held firmly together by a girth of four tension members, is a human invention and probably thousands of years old. Long before people covered it with paper for use as a flying object to loft in the wind the frame most likely was used as a lightweight pallet, a stretcher for transporting things. Basic as it is, the prestressed kite exits only in the world of people; not as a product of nature.

The kite frame can be built in many proportions as shown below. The structural principle remains the same except that the distribution of forces, both tension and compression, vary as the proportions are altered. Always, though, the total of the compression forces pushing out are equal to the sum of the tension forces pulling in.

The kite frame is quasi-tensegrity because the two struts, lacking a force in the “z” direction in order to separate, touch and press on one another where they cross. The kite structure is the basic prestressed tension-compression cell for x-module tensegrity structures.

The lengths of the four tendons and the lengths of the struts determine the shape.

The kite frame’s tension and compression system. The struts push out (compression) and the tendons pull in (tension).

Kite frame shape variations
The kite frame is transformed into a true tensegrity structure with a third strut which is added by replacing one of the original edge tension lines (in green) with four new lines shown in red. These four perform as slings that suspend the new strut.

The three-strut structure must now be made stable by adding two additional lines - draws -- those shown in blue. The draw tendons go from the ends of the new, third, strut to the far ends of the original pair; to those ends that will draw the kite struts away from one another. Connected to the wrong two ends the draw lines will only force the kite struts into firmer contact and fail to achieve a floating compression structure.

It is essential in this simple structure, as in all tensegrity structures, to establish the optimum lengths for the tension members so that the work will be firm and tightly prestressed. This can only be done by successive adjustments, by trial and error. If the length of one line is changed the tension on all lines are effected.

As a general rule the draw lines are the primary means for increasing or relaxing the prestressing of a tensegrity structure. As with most rules, there are a variety of exceptions.

This construction process, adding parts one by one, has now transformed the basic kite frame into a proper three-strut tensegrity module. It is structurally the same as the three-way module shown on page 15. Only their shapes and symmetries are altered by size adjustments of the tendons and struts.

**Replacing A Single Strut With An X-Module**

In the figure to the right, the third strut that was introduced in the figure above is itself replaced by a second kite frame; slings in red, draws in blue. This new assembly of two “x-modules” represents the first step in a construction process -- adding module after module -- that can be expanded indefinitely. Each open quadrant of any module offers a place to connect yet another x-module.
The term *space-filling* applies to such systems, for example, as sugar cubes packed in a box or oranges at the market. The X-module too can be expanded, adding module after module, in all directions. Below are examples of how “X” modules can be repeated indefinitely.

*Double Star 1950-2002*

An expanded X-module plane

X-module 90 degree corner is created by adding a third module.
TENSEGRITY ADAPTS TO A VARIETY OF FORMS

These sculptures are composed of six struts
THE BINARY GEOMETRY OF MAGNETS
A unique group of five polyhedra have a special quality: they permit the checkering of adjacent faces. This natural binary property makes it possible to construct them using polygon-shaped refrigerator magnets whose north and south poles are on opposite faces like heads and tails of a coin. When assembled the magnets snap together edge-to-edge to create a firm magnet polyhedron.
MAGNETIC ARCHITECTURE

Eight-magnet octahedra and fourteen-magnet cuboctahedra can be assembled together to form an expandable space-filling matrix with binary magnetic bonding from cell to cell.
SPHERICAL GEAR TRAINS, MAGNETIC AND MECHANICAL

Pictured below are two sets of gear trains. In the upper row the gears are made of disc-shape magnets with north and south poles on opposite faces like the magnet-polyhedra on the previous two pages. These spherical binary mosaics have the peculiar number: 5, 8, 10, 14, 18 and 32.

The disc-shape magnets in the top row are ceramic magnets supported on non-magnetic armatures. They snap together edge-to-edge through north-south attraction in the same manner as the polyhedron magnets but because they are round and in magnetic contact they can revolve as sets of gears. If one magnet is made to revolve by hand the others follow as gears.

In the bottom row are computer images of metal gears that have the same numbers and geometry as the magnet spheres of the top row.

Mechanical or magnetic, all of these binary phenomena are first principles rules of fundamental geometry; laws that determine the design of structures in nature.
Clockwise and counterclockwise rotation, a binary principle as well as a symmetry principle, can be seen in many ways in various structures. When I use the word “rotating” to describe the order of threads in a weave pattern or the arrangement of struts in a tensegrity structure it is clear they are in fact sitting still. Inside of the structure though, the forces are acting in either a clockwise or counterclockwise direction. Even so, this helical tendency is translatable into actual motion by transposing the static domains of tensegrity figures into actual wheels or gears. Shown here are three tensegrity structure examples along with sets of disk-shaped magnets. These spherical arrangements are born of the same geometry as the tension networks of the tensegrity structures.

As with the polyhedral magnet mosaics the disc magnets shown here have their north poles on one face and south on the other so that when they are edge-to-edge, checkerboarded with opposite poles facing out, they snap together. Rotating one magnet by hand causes the others follow as a gear-train.

The magnet structures are a part of my multimedia, artwork, “Portrait of an Atom”. They also tell us that the world of structure and geometry is a hall of many mirrors endlessly reflecting similarities, relationships and numbers. In the case of the magnets, the directions the arrows point describe not only the rotation and counter-rotation of the gears but they also identify the direction electrons would be moving if these were current electrical loops rather than permanent magnets, in order to produce north/south magnetic attraction. Though my atom model, “Portrait of an Atom” is speculative, the magnet relationships and their geometry are a fact of nature.
EXTENDED MAGNETIC GEAR TRAINS

These magnet assemblies, like the individual magnet spheres, are unit gear trains: one of the magnets is made to turn by hand the rest will follow in unison. I discovered the magnet structures over fifty years ago, in 1962. From that discovery was born my fascination with the atom’s electronic architecture.

A body-centered cubic arrangement of 14-magnet spheres. Each sphere connects to its neighbors at the corner positions of a cube.

A body-centered array of 8-magnet spheres. If this pattern is extended indefinitely each 8-magnet unit will have 8 neighbors at its corner positions.
5-magnet cells in a hexagon beehive pattern. Magnetic linkage is continuous. In my atom model, this hexagon formation represents the arrangement of carbon atoms in a plane of graphene.

A cubic form composed of 8-magnet spheres alternating with 14-magnet spheres in perfect magnetic continuity. The polarities of the adjacent cells have reverse polarity. If a 14-magnet sphere has its 8 corner-magnet south poles facing out and its 6 face-magnet’s north poles facing out, its neighboring 14-magnet set will orient these polarities in reverse.
These principles of structure are the foundation of my model of the atom. See these interesting papers:

PDF atom files at kennethsnelson.net/the-atom
http://kennethsnelson.net/PortraitOfAnAtom.pdf
http://kennethsnelson.net/SnelsonAnArtistsAtom.pdf
http://kennethsnelson.net/Snelson_CirclesSpheresAndAtoms.pdf
http://kennethsnelson.net/articles/KSnelson_paper_FQXi_updated.pdf