CIRCLES, SPHERES AND ATOMS

Kenneth Snelson

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"A Design for the Atom," Industrial Design Magazine, Feb., 1963
“Discontinuous Compression Structures”, Feb., 1965 U.S. Patent #3,169,611*
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Kenneth Snelson,“Portrait of an Atom”, 1981, Maryland Science Center, Baltimore, Maryland, U.S.A.
Exhibition, “The Nature of Structure”, N.Y. Academy of Sciences, 1989
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Abstract:

The author describes his four-decade interest in the geometry of small nonoverlapping circles on spheres and their relation to normal polyhedra. He discusses the varieties of circle-sphere geometry and their association with mosaics of circle-shape permanent magnets on spheres. He outlines the stages in developing his long-running, open-ended, artwork, an analog model of the atom’s electronic architecture based on circlesphere geometry.

> “If you have had your attention directed to the novelties of thought in your own lifetime, you will have observed that almost all really new ideas have a certain aspect of foolishness when they are first produced.” Alfred North Whitehead:

Standard polyhedra, tetrahedra, cubes, octahedra... have a sister variety of hedra whose faces are circles instead of polygons. I call these interesting forms that lack a customary name, “circlespheres”. The occasional references I have found for circles on spheres in mathematics are concerned with the problem of how, most economically, to pack a number of equal nonoverlapping circles on a sphere.

Circlespheres have led me, as an artist, into a quite different path from that of mathematicians. My fascination from the beginning, going back forty years, was with the visual and tactile experience of exploring the many varieties of circlespheres and, finally, with an intense period of studying the history of models of the atom, I began working on what has become a complex multimedia artwork, “Portrait of an Atom”, whose geometrical order is of this fascinating circle geometry.

My study began not from any special interest in the mathematics of circles and spheres but rather from exploring the straight-line geometry of my steel tube and cable sculptures. They were, in 1960, an unusual and new type of structure I had discovered twelve years earlier and called “Floating Compression”. The engineer/architect Buckminster Fuller later made up the name that has stuck: “tensegrity” from tension and integrity.

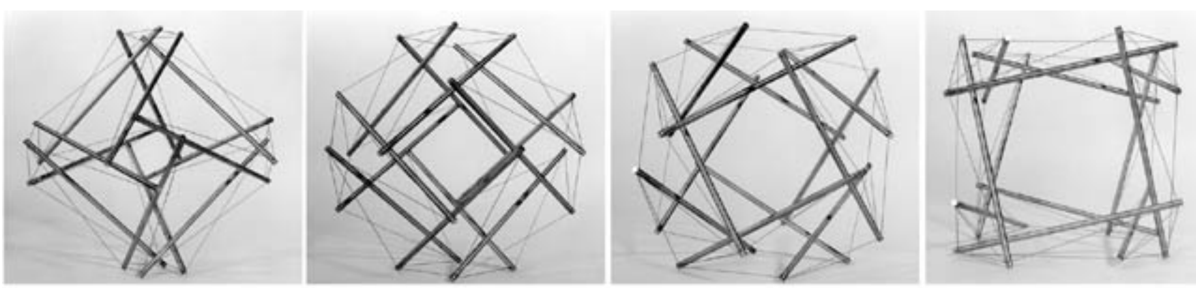
To define tensegrity is difficult because it has become a buzzword for any object or architectural idea that includes visible tension wires. In my art, tensegrity refers to a lightweight modular structure of three or more compression struts pushing out against a closed network of nonredundant tension wires. The whole system is so arranged that the struts contact only the external prestressed tension network, not one another. (Fig. 1)

Kenneth Snelson, “Easy Landing”, Baltimore, Maryland, U.S.A.

I was working in my studio all that year, 1960, building models to learn more about the many tensegrity forms, their modular properties and the puzzle of putting these complicated, often frustrating, structures together. It was an especially exciting time because I knew it was not likely that anyone anywhere had studied the floating compression principle before. I did consider that in some ancient day, in China perhaps, a scholar had traveled the same path, maybe constructing objects out of silk cord and bamboo sticks. If so the record was not to be traced in everyday sources.

In order to move on from one discovered form to another, deciding what to do next, I often asked "what if". One such question came from noticing the changes that were possible in a single configuration if I altered the lengths of tension members. Put it this way: because the tendons can be lengthened or shortened relative to one another thereby changing the form while still maintaining a stable structure, I might construct a series of transformations and then photograph them in sequence and see the changes in animation.

For example, to see the way the octahedral12-strut tensegrity form can transform into cube, I constructed just four stages of the mutation. To do it properly would have required a dozen or more in-between stages. Nonetheless the propeller-spinning of each strut could be clearly seen from one stage to the next. (Fig. 2)



Tensegrity 12 strut octahedron with passages to 12 strut cube

This study of spinning rods had distracted me from tension lines and compression struts to focus on the implied cyclical motion within the forms. These traces would of course be circles. In an effort to describe the rotations I first made cardboard disks and taped them together but the results were inelegant and they did not truly define the sticks' motion planes. They did awaken my interest in looking seriously at hedra composed of circles.

Most opportunities in life arrive by accident. My circle-motion fascination would soon have faded had it not been that my studio was but a short walk to New York's Canal Street, which in those years was still a place to go rummaging through stores stocked with odd industrial mistakes and manufacturing surplus items. With my mind fixed on rings and circles I discovered at "Industrial Plastics" a big bin of thin bracelet-like plastic rings. They were cream-colored, semitranslucent and mixed 3, 4 and 5 inches in diameter.

Suddenly from puzzling over what kind of circle objects -- washers? large keyrings? -- I might use to construct rings on spheres, I had stumbled onto a supply of perfect rings, more than I possibly could have imagined, ten for a dollar. Seeing this as a rare opportunity I bought three hundred on the first visit and the next day, fearful they would be disappear, I returned to the store and at a very special price emptied the entire bin of somewhat over five-hundred rings.

Back in the studio, I devised a jig for drilling 2, 3, 4... tiny holes in the edges of the rings at the contact positions the particular form required. I began tying them hole-to-hole with nylon fishing line and tight square-knots to create the various circlesphere cages simply from deducing what symmetry choices were likely.

I set only these initial rules, virtually the same ones applied by the mathematicians investigating the problem of economical packing.

1. Non-overlapping small circles must be of the same size on a sphere.

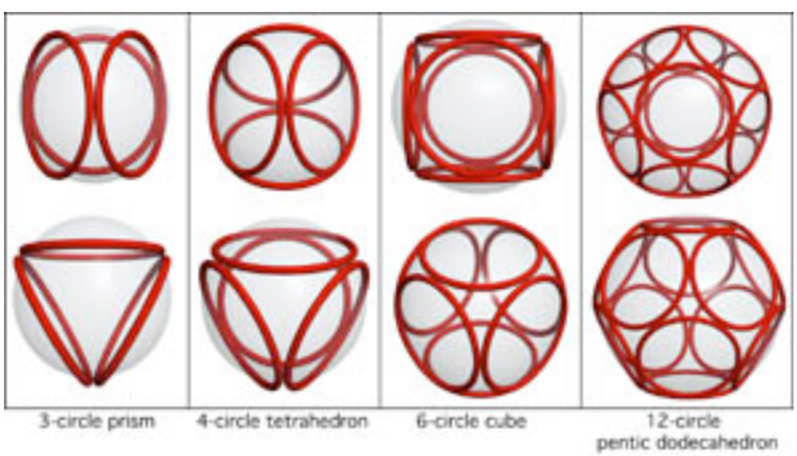
2. The space between the circles, that is the interstitial spaces, must be smaller than a circle's diameter; open nets are disallowed.

Over the following days and weeks I drilled hundreds of holes and with worn and raw fingers I tied hundreds of knots in creating what became an impressive assortment of circlespheres and their spaceframe matrices. In the process of constructing them I quite fell in love with the beauty and order of these unusual objects (Fig. 3 Many circlesphere configurations.)

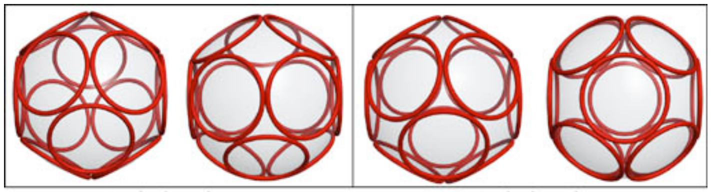
Some of the unit cells are clearly familiar polyhedra except with circle faces but others are uniquely circlespheres with no polyhedron equivalent.

In normal polyhedra, the faces take up the total surface. There are no inbetween spaces. In circlespheres the opposite is true: the gaps between circles are concave polygons; triangles, squares, pentagons or hexagons.

Four of the circlesphere forms have triangle interstices only: A 3-circle prism has, top and bottom, two triangle openings. The 4-circle tetrahedron has triangles at the 4 vertex locations. A cube made of 6 circles has 8 triangles at the corner positions. A 12-circle pentic-dodecahedron has 20 triangles at the icosahedral positions. (Fig. 4)



A few circlespheres such as the 12-circle rhombic and the 12-circle trapezo-rhombic dodecahedrons have both triangle and square interstices. (Fig. 5)



12 circlesphere
rhombic dodecahedron

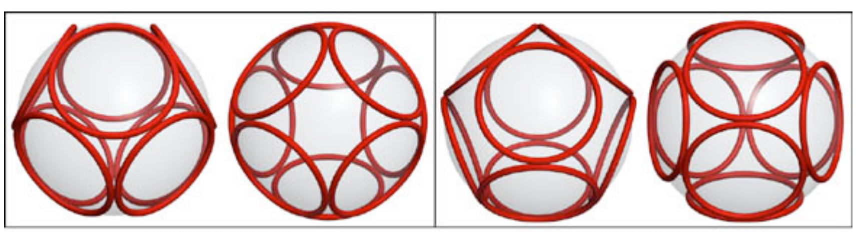
12 circlesphere
trapezo-rhombic dodecahedron

Most interesting though are the figures whose open spaces involve an even number of circles, 2, 4 or 6. These sets can be checkerboarded with 2 colors so that no rings of the same color contact one another.

There are 7 such sets. The list includes the simplest of all: 2 circles in contact straddling the equator. The 2 rings share a common face-to-face interstice but they are assigned opposite colors.

The checkerboard cages are those with 2, 5, 8, 10, 14, 18 and 32 rings. (Fig. 6)

The 8-circle checkerboard arrangement is octahedral. The 2 other 8-circle figures, an antiprism and a curious figure I call a “bird house”, are composed partly of triangular interstices so they do not checkerboard. (Fig. 7)



8-circle anti-prism

8-circle “birdhouse”

Among the 7 binary cages, only the figure with 18 circles, a rhombicuboctahedron, has open spaces comprising 6 rings and is the least sturdy of all the circlespheres.

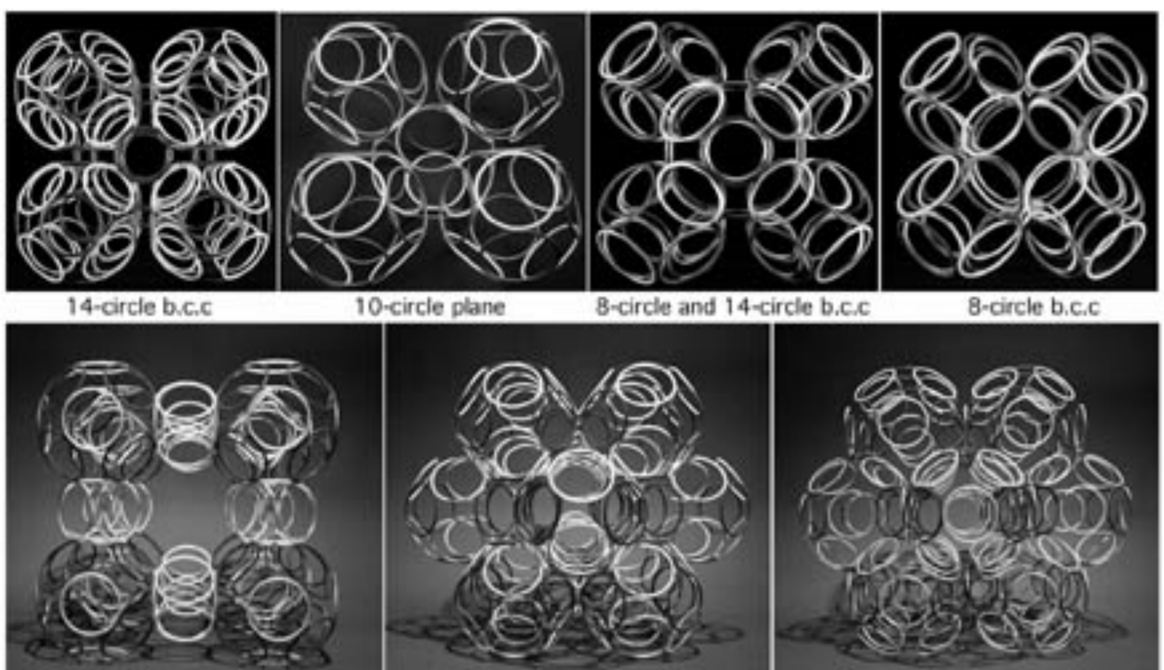
Because my plastic circles included 3, 4 and 4 inch rings I experimented briefly with the possibilities of using different size rings on a sphere but there seemed to be no way to be certain the varied rings were forming a true sphere. Clearly one might apply any number of smaller and larger circles on a big sphere and arrive at many orders of complexity.

I did discover an interesting fact about the 4-circle tetrahedral sphere: that any size-combination of four rings in tangent contact assembled as a tetrahedron will define a true sphere. I constructed two demonstration devices to illustrate this principle in different ways. (Fig. 8)

#1 The four plastic rings are fixed at their edges to form a tetrahedron. Each one has a flat vinyl disk and a cone for centering a sliding pointer-rod. The tips of the four rods touch at the center showing that the rings lie on a common sphere.

#2 The tetrahedron's four metal axial rods are loosely joined at the center. Each plastic disk is able to slide firmly along the rod. When the disks' edges meet at tangent points they have formed a sphere.

After a few months of tying rings together, photographing them and deciding what to do next, I could look about my studio and see a dense garden of skeletal cells and matrices, including special sets with checkerboard patterns throughout. (Fig. 9)



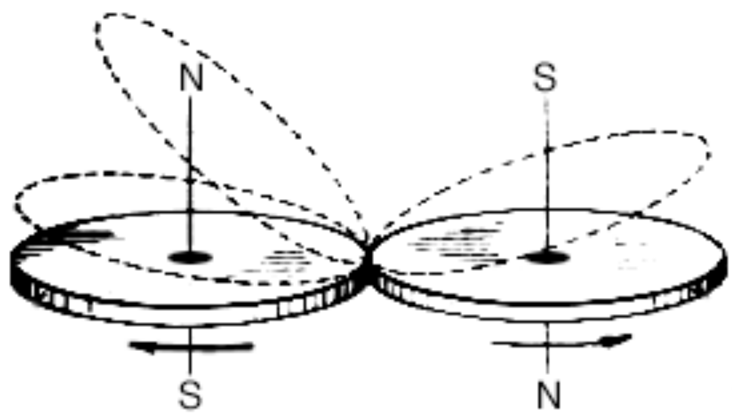
14-circle b.c.c — 10-circle plane — 8-circle and 14-circle b.c.c — 8-circle b.c.c

14-circle and 6-circle spheres cubic packing — 14-circle spheres face-centered cubic — 18-circle spheres face-centered cubic

These many cages sat on every available surface; additionally many hung from the ceiling. As the numbers grew I was endlessly delighted to reflect on my good fortune of finding Industrial Plastic's surplus rings, uninteresting to passersby who saw only a bin-full of surplus junk, whereas they had now become a jewel-case of surprising geometrical objects.

It was in May of 1961 I was buying nylon line at a hardware store when my eye fixed on a rack with magnetic refrigerator towel hooks. They were round. Thinking always of the circlespheres and the binary mosaics, I had a flash of association, a link connecting binary checkerboard spheres and the physical binariness of north and south magnetic polarities. Chance and fortune do indeed favor the prepared mind. My mind was on the checkerboarded 8 circles of the octahedral circlesphere so I bought 8 of the magnetic hooks.

Back at the studio I excitedly pried the magnets out of their steel casings. Yes, they were round disks made of ceramic magnetic material and they measured 30mm in diameter by 3mm thick. At the center of each magnet was a 5mm hole; an important detail. I stacked up the 8 magnets with all poles in parallel, that is all facing the same direction and they stuck tightly together. If I tried to reverse 2 or 3 of them they repelled with equal force. Placing one next to another lying on the table I verified that they were magnetized through the flat surface with north and south poles on opposite faces like heads and tails of a coin – or, as I later learned, like the magnetic field of a current loop of electricity. (Fig. 10)



With 2 magnets lying flat next to one-another if both had their north poles facing up, their parallel edges repelled one another. If I flipped one magnet over, their edges clicked together: magnetic antiparallel. So that's the way their fields of magnetism work.

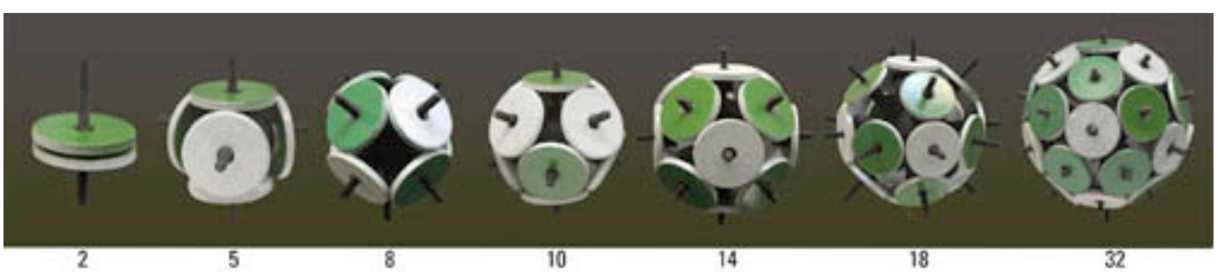
Could they be made to connect successfully if placed in the proper positions of the 8-circle sphere? I tried in every way possible to hold them and make them stand up to form an octahedral circlesphere but they simply collapsed in a jumble. Apparently they needed to be somehow mounted in position. I decided to make an armature to support them in the octahedron's face locations. Using a plastic marble about 25mm in diameter I bored and tapped eight holes to receive brass threaded-rod, non-magnetic, as posts to fit the magnets on.

Setting each one on its own axle, north to south, north to south, they magically clung together edge-to-edge on the armature, forming a perfect octa-circlesphere, astonishing how they linked one-to-another. I found that if I held any one of the 8 with thumb and forefinger and turned it like a wheel, surprisingly, the entire set followed along, an 8-gear differential. (Fig. 11)

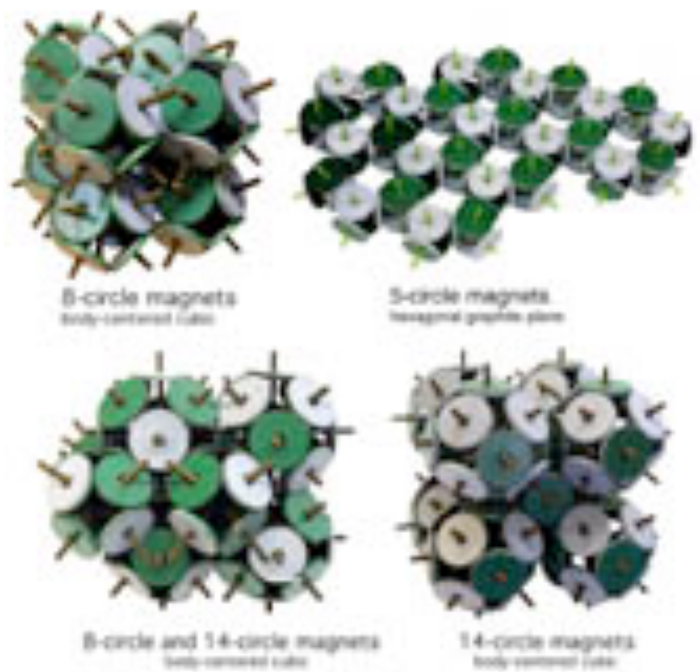
8 magnet circlesphere

No single learning event in life has come close to the elation and wonder I felt at that moment, discovering a quite unexpected relationship, straight from the core of the cosmos, so it seemed to me: a marriage of natural binary principles. The first, a fundamental mathematical property of symmetry points on a sphere; the second, the simple bipolarity of north/south magnetism. Clearly the 8-circle magnet cell was but one of seven checkerboard circlespheres that could perform in the same remarkable way.

I ordered a quantity of magnets from the manufacturer and constructed the remaining 6 checkerboard spheres, (Fig. 12)

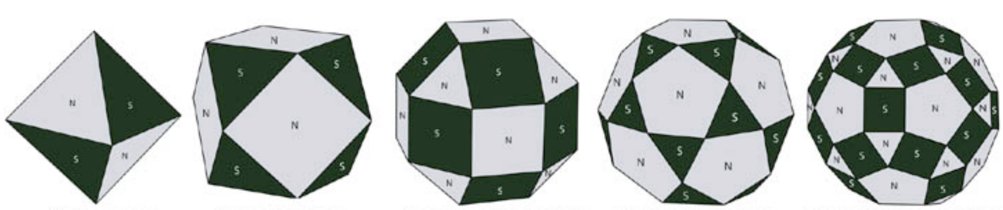


going on then to make examples of magnetic spaceframes in which the north/south association of the individual cells are continuous through an endless repetition of magnet spheres. (Fig. 13)



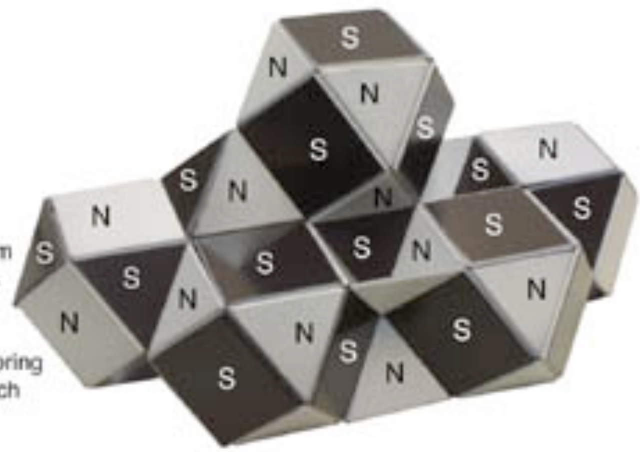
If instead of permanent magnets these matrices were composed of current loops the electrons traveling in the wires would also be moving in the same clockwise/counterclockwise three-dimensional, endless, chain.

Years later when I could finally think far enough distance from my circlesphere occupation I made what seems now an obvious extension of the magnet mosaics: The checkerboard principle works just as well with certain normal polyhedra by using flat polygon shaped magnets polarized on opposite faces. The polyhedra that can be checkerboarded include the octahedron, the cuboctahedron, the rhombicuboctahedron, the icosidodecahedron, the rhombicosidodecahedron, and others. Like the circle magnets some of the magnet polyhedra can translate indefinitely as endless magnetic space frames. Unlike the circle-magnet spheres these systems are self-supporting without armatures. (Fig. 14)



Octahedron Cuboctahedron Rhombicuboctahedron Icosidodecahedron Rhombicosidodecahedron

A cuboctahedron/octahedron binary construction assembled from square and triangular flat magnets with north and south poles on opposite faces. The magnets fasten edge-to-edge when neighboring polarities are antiparallel to form rich varieties of magnetic architecture.

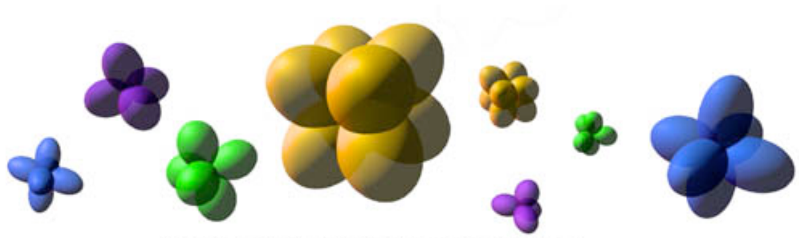


(Fig. 15)

It may seem odd to some that this magnet adventure led to thoughts about atoms. But consider the fact that atoms are spheres, that atoms bond to one another at geometrical angles, that electrons within the atoms are the source of the active magnetism in the permanent magnets, that electrons fill shells in discrete numbers. And here in the magnet circlespheres was a curious coincidence: the numbers of magnets in the checkerboard spheres: 2, 5, 8, 10, 14, 18 and 32, were so near to the textbook list of shells and subshells of electrons: 2, 6, 8 ,10, 14, 18 and 32. Was it not reasonable for an inquisitive mind to consider an analogy with atoms and their electrons?

Those who have studied even a limited amount of physics and also those who have read about the quantum atom and the quantum revolution which began at the first part of the last century know that science does not know what an atom would look like if we could magically shrink down to the submicrocosm and try to see it -- nor how the atom's electrons do their work to create its amazing performance as a tiny electro-mechanical device. For this reason all atom models are inventions of the mind, based on some kind of analogy from the visible world.

The physicists' earlier models from 1900 to 1924 used various analogies including a culinary one with J.J. Thomson's raisons-in-pudding model. The final one of course is the "charge cloud" model whose electrons' presence have been likened to vapor. (Fig. 16)



Schroedinger charge-cloud atoms

The best known physical analog is Neils Bohr's planetary model. It was clear that for me to invoke the circlesphere geometry and magnet spheres to represent electrons moving about was, from a physics point of view, lunacy. First, of course, atomic physics had abandoned the Neils Bohr conception of electron orbits many years ago. Even worse than planetary electron orbits, my thoughts of electrons traveling in small circles off-center from the nucleus was the equivalent to imagining an Earth satellite, on top of the globe, circling the north pole.

Still the urge to try was irresistible and I had the advantage of coming from the outside, never having studied quantum physics. It was only through wishing to resolve these dreams of geometrical atoms that I begin to go deeply into the subject of atom models and to start collecting a fine library of atomic physics and the history of the atom from the time of Democritus and Leucippus in ancient Greece.

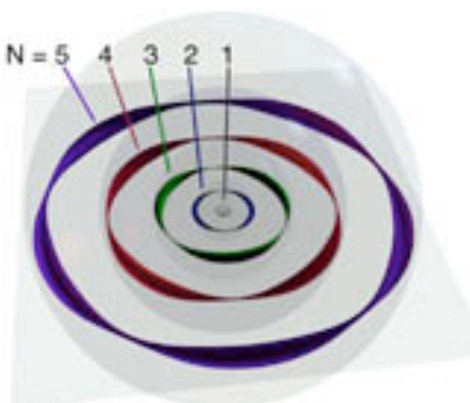
By 1930 atomic physics began prohibiting models that attempted to describe the workings of the atom's electrons. The last such successful physical atom model arrived in 1924 -- the French physicist Louis de Broglie's model adapted from Bohr's 1913 planetary picture. There have been no other recognized ones since that time. It seemed best therefore for me to begin where Louis de Broglie's model stopped – or was left incomplete. It was he who first imagined that material objects might have a wave aspect, mirroring Einstein's discovery that radiation, light waves, have a particle aspect: photons. His theory is also at the root of the Schroedinger wave equation, although transformed from its original physical sense into mathematical formalisms.

De Broglie pictured the hydrogen atom's lone electron, not as a tiny missile racing around the nucleus as Neils Bohr had done, but as a circle-shaped standing-wave like a vibrating guitar string surrounding the nucleus. Like Neils Bohr's particle electron, his matterwave electron inhabits only certain restricted orbits on electrical spheres, quantized in steps like notes on a piano. In order to move to a larger or smaller sphere (or energy level) the electron was required to perform electrical work, taking in or giving off specific amounts of energy, photons -- much like going up or down stairs.

This image offered a logic for my circlesphere proposal: For while Bohr's planet-electron could not circumnavigate a small-circle orbit, why should a matterwave be prohibited from doing so as long as it remains on the same electrical sphere? My starting point, like de Broglie's, is that electrons act not like missiles, but like waves and that they are genuine material items.

But I also include this: that each electron's orbit has a matter-like barrier property by which it occupies its individual space, a circular shield that enables it to exclude others from its piece of atomic real estate. This proprietary property is reasonable physical interpretation of Wolfgang Pauli's exclusion principle

In contemporary textbooks one finds only a terse presentation of the de Broglie model, describing that at each successively larger circle surrounding the nucleus the electron includes an additional whole wave: The smallest orbit called the *ground state* has 1 wave. The second shell allows 2 waves, the third level, 3 etc. (Fig. 17).



Neils Bohr-Louis de Broglie model
of the hydrogen atom

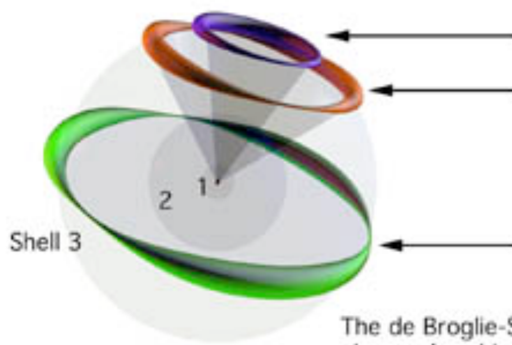
In a careful examination of de Broglie's model I saw a curious property not really hidden, but certainly never commented on; one that is inherent in the Bohr analogy of the electron as a planet, acknowledging that planets in more distant orbits travel slower than those closer in. Similarly, the electron in the Bohr-de Broglie model slows down in its larger orbits with a resulting increase in the length of the wave. But increasing by how much? Remarkably, in its modular quantized way, the atom lengthens the electron's orbital circle at each successive shell by the length of the ground-state's circumference. Shell two's 2 waves, for example, are each twice as long as the ground state's wave. Its two waves then make the orbital circle 4 times longer. Shell three's waves are 3 times the ground state's wave making its circle 9 times as long. Shell four's waves are 4 times the ground state wave making its orbit 16 times as long. (Fig. 18)

de Broglie model with wavelength increases at each shell by the ground state's orbital circle

If indeed anyone ever has noticed this odd geometrical fact the likely response was, "so what?" But to me it suggests how the electron uses quantization in a very special way. At the very least it reveals that at each shell the electron assumes a specific wavelength, vibrating like a musical tone, a note to be found only at that sphere and at no other within the atom. It becomes the basis in my model for proposing that the electron is always on automatic pilot and that its wavelength is keyed to a specific energy sphere; an interlock. The electron's altitude dictates its wavelength and vice versa.

This becomes useful in a significant way -- for providing the electron its required optional orbits for each shell, normally designated "s" electrons "p" electrons, "d" electrons, etc., names left over from 19th century spectroscopy. For example: the third shell is required to have three options: 3s, 3p and 3d. Its normal 3s de Broglie orbit, the equatorial state, has three waves.

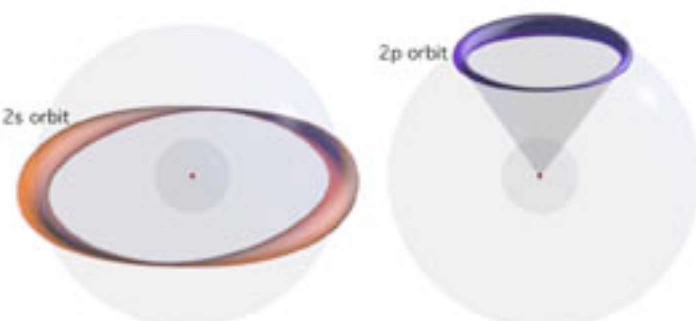
According to my model when the orbital wave surrounding the equator is pressured by its neighbors it can no longer control that prime space. Crowding on the shell will demand that it confine itself by folding its three-waves into two, a snake eating its tail. Squeezed in this way the orbit cannot encompass the equator but it remains on the same electrical sphere (3p state) in a small circle whose circumference is two proper 3rd shell wavelengths. With even greater crowding, the matterwave can retract into its ultimate confinement, a one-wave, 3d state. It now completes each revolution in a third the time of the equatorial orbit. (Fig. 19)



The de Broglie-Snelson hydrogen atom electron's orbital options on shell 3

By this process the 4th shell will have four options (4s, 4p, 4d and 4f), the 5th shell five, etc. Electrons compressing one another is not the only reason non-equatorial orbits occur. They also happen in my model when incoming light lifts the electron momentarily from the ground-state, for example, to a one-wave state in a higher shell.

Thus, the circlesphere geometry converts the Bohr-de Broglie pancake atom into a fully three-dimensional one. The familiar textbook pictures of the Schroedinger balloon-lobes reaching out from the nucleus provide the electrons hands and arms -- like ball-and-stick lab models -- for connecting atom-to-atom. This directional, geometrical, need is satisfied in the circlesphere atom's orbits which, while remaining on their own energy shells, still project from the nucleus into the surrounding space for bonding. (Fig. 20)



De Broglie 2nd energy level's original 2-wave orbit.

Snelson model's 2nd energy level 1-wave orbit with cone indicating its direction in space in relation to the nucleus.

In my view, the standard model's awkward geometrical solution is analogous to the planetary epicycles of Ptolemy, mathematically correct but physically untenable. Circlesphere geometry, eminently adaptable for the purpose, has been waiting in the wings until a likely mechanism might arrive to turn it into an atom.

In brief, then, here is my artwork atom: The electrons are circle-shaped standing-wave objects. Each matterwave is a perpetual, friction-free, disturbance on an equipotential sphere. The sphere they ride on is real only through the electrons' presence. Without them the protons' electrical field is but a force radiating from the nucleus but without punctuation.

The electron orbits are, themselves, tiny devices equipped with 5 distinct forces through which they interact with one another:

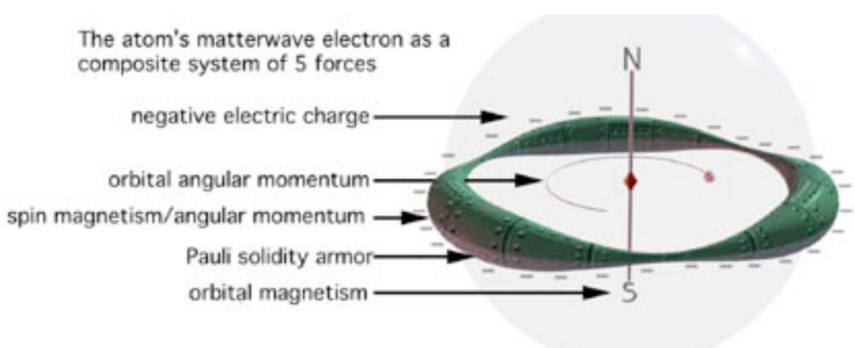
1. De Broglie's matter waves posses a barrier property enabling them to exclude one-another. Thus the solidity of material objects begins at the atomic level with the individual electron. While electrical attraction to the nucleus is the atom's tension strength, the matterwave's solidity is its compression resistant strength. All structures need both tension and compression. Matterwave solidity is a genuine force because when pushed against it pushes back.

2. The particle electron's negative charge. It is the electrons' attraction to the nucleus that binds them to the atom: a tension force. In a normal atom electrons and protons neutralize electricity. In orbit this negative field is evenly distributed throughout the circular matterwave.

3. Spin: an intrinsic magnetic and top-like property of the particle electron. Traveling within the wave, the electron's spin can be in the same direction as the orbit or by inverting it can "spin" in the opposite direction.

4. A second magnetic field, orbital magnetism, arises out of the particle's electrical charge circling in orbit. Orbital magnetism can be either enhanced or diminished by the orientation of the electron's spin; an energy-conserving toggle.

5. The electron's tiny mass revolving in orbit gives rise also to a gyroscopic angular momentum that adds to the wave-orbit's stability.



(Fig. 21)

As individual torus modules the matterwave orbiting electrons are the atom's building blocks. They keep one-another out but they can adhere through magnetic attraction. In constructing an atom the electron waves fill up the shells one-by-one and when a shell becomes crowded, the next members start a new shell. Magnetism plays a significant role in designing the shell's circlesphere configuration. (Fig. 22)

Snelson model of a heavy atom with many electrons has numbers of shells within shells around the nucleus, each composed of electron matterwave orbits.

In the same way that water in a pond seeks its own level, electrons, too, arrange their assorted forces to maintain the atom at its minimum energy state. Though they are but pseudo objects, ethereal pathways of perpetual motion, they are the atoms-within-the-atom that give rise to its architecture. They are the substance of all the things we know as matter, from a puff of steam to the hardness of a diamond.

My atom model has been shown in art museums as well as science museums and, as a means for publishing, I hold two U.S. mechanical patents defining the model's main magnetic/geometric properties. On the web a search engine will turn up numerous links from educational websites to "Snelson Atom".

Again, this is not a work of science but a work of art. Even so, over the years I have enjoyed discussing my atom model and corresponding with many scientists including Linus Pauling, Richard Feynman, Philip Morrison, Eugene Wigner and Hans Christian von Baeyer. None have given it a ringing endorsement. One physicist said, "Just because it's beautiful doesn't make it right." Another said, "It's just not the way an atom ought to look."

Science writers advise us over-and-over that the instant we reach down into the quantum world we are entering a realm so unfamiliar, so strange, that we should not expect things to make any sense. It is this self-fulfilling belief which has, in my opinion, kept scientists from arriving at a physical model on their own. The choice of appropriate geometries is limited and it is certain that when the atomic physicists' curious world-view concerning the atom swings in another direction, as attitudes do over time, the phenomenon of circlespheres and their association with magnetism will be discovered as a rich field to explore.

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